3.1a: Derive equation 3.16 from the binomial distribution and stirling's approximation

$$p(x) = e^{-N} N^x / x!$$

$$p(x) = e^{-np} (np)^x / x!$$

$$p(x) = \left(\frac{n!}{(n-x)!\,x!}\right) p^x (1-p)^{n-x}$$

Expand factorials

$$pn(x) = \left(\frac{\sqrt{2\pi}\, n^{n+1/2} e^{-n}}{\sqrt{2\pi}\,(n-x)^{(n-x)+1/2} e^{-(n-x)} x!}\right) p^x (1-p)^{n-x}$$

Eliminate $\sqrt{2\pi}$s and rearrange

$$\left(\frac{n^{n+1/2} e^{-n}}{(n-x)^{(n-x)+1/2} e^{-(n-x)} x!}\right)\left(\frac{p}{1-p}\right)^x (1-p)^n$$

$$(1-p)^n = \left(1 - \frac{N}{n}\right)^n \rightarrow e^{-N}$$

$$li\, m_{p\rightarrow 0} \left(\frac{p}{1-p}\right)^x = p^x$$

$$\left(\frac{n^{n+1/2} e^{-n}}{(n-x)^{(n-x)+1/2} e^{-(n-x)}}\right) p^x e^{-N} / x!$$

$$e^{-np} (np)^x / x!$$

3.1b

3.2 How many photons must be detected to determine rate within :

1%: $.01 = \frac{1}{\sqrt{N}} \rightarrow N = 10{,}000$

$\frac{Nhc}{\lambda} = 4fW$

1ppm: $10^{-6} = \frac{1}{\sqrt{N}} \rightarrow N = 10^{12}$

$\frac{Nhc}{\lambda} = 400nW$

3.3a

Johnson Noise: $\langle V^2 \rangle = 4kTR\Delta f$ = $4k(300K)(10{,}000\Omega)(40{,}000Hz)$

$$2.57\mu V$$

Input Voltage 20dB hotter = 10* ($4k(300K)(10{,}000\Omega)(40{,}000Hz)$) =

$$25.7\mu V$$

3.3b

Capacitor Fluctuation: $\frac{1}{2}CV^2 = \frac{1}{2}kT$

$$C = kT/V\text{^}2$$

$$C = 625pF$$

3.3c

Shot Noise (Current)^2 = $2q\langle I \rangle \Delta f$

$$\left(1\% \frac{25.7\mu V}{10k\Omega}\right)^2 \frac{1}{2q40{,}000Hz} = 51.5nA$$